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CORRELATIONS PREDICTING FRICTIONAL PRESSURE DROP AND LIQUID HOLDUP DURING HORIZONTAL GAS–LIQUID PIPE FLOW WITH A SMALL LIQUID HOLDUP

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Abstract—Experimental data and correlations available in the literature for the liquid holdup ϵ_L and the pressure gradient $\Delta P_{TP}/L$ for gas–liquid pipe flow, generally, do not cover the domain $0 < \epsilon_L < 0.06$. Reliable pressure-drop correlations for this holdup range are important for calculating flow rates of natural gas, containing traces of condensate. In the present paper attention is focused on reliable measurements of ϵ_L and $\Delta P_{TP}/L$ values and on the development of a phenomenological model for the liquid-holdup range $0 < \epsilon_L < 0.06$. This model is called the "apparent rough surface" model and is referred to as the ARS model. The experimental results presented in this paper refer to air–water and air–water + ethyleneglycol systems with varying transport properties in horizontal straight smooth glass tubes under steady-state conditions. The holdup and pressure gradient values predicted with the ARS model agree satisfactorily with both our experimental results and data obtained from the literature referring to small liquid-holdup values $0 < \epsilon_L < 0.06$. Further, it has been shown that in the domain $38 < \sigma < 72$ mPa m the interfacial tension of the gas–liquid system has no significant effect on the liquid holdup. The pressure gradient, however, increases slightly with decreasing surface tension values.

Key Words: gas–liquid pipe flow, liquid holdup, pressure drop, small liquid holdup, stratified–wavy flow regime, annular flow

1. INTRODUCTION

For a quantitative description of gas–liquid flow in pipelines, it is of importance to understand the phenomena in this kind of flow. Despite the numerous theoretical and experimental investigations on gas–liquid pipe flow, no reliable frictional pressure gradient and liquid-holdup correlations are available, mainly as a result of the large number of variables involved.

Some authors (e.g. Storek & Brauer 1980; Olujic 1985; Gregory *et al.* 1985; Battarra *et al.* 1985; Müller-Steinhagen & Heck 1986) have attempted to find general models for the frictional pressure gradient and liquid holdup in two-phase pipe flow by curve fitting on existing data. A disadvantage of this approach is that the correlations obtained in this way are strongly dependent on the composition of the databanks used.

Generally, better agreement between experiment and theory is found when phenomenological models are used for the description of gas–liquid pipe flow. Most of these models have been applied to *stratified* flow (e.g. Cheremisinoff & Davis 1979; Chen & Spedding 1981; Kadambi 1981), *annular* flow (e.g. Hoogendoorn 1965; Wallis 1970; Hashizume 1985) and *slug* flow (e.g. Dukler & Hubbard 1975; Kordbyan 1985).

In a previous paper Hamersma & Hart (1987) presented a phenomenological model to calculate the value of the frictional pressure gradient in gas–liquid pipe flow with a small liquid holdup ($\epsilon_L < 0.04$) covering the stratified, wavy and annular flow regimes. A disadvantage of a phenomenological approach is that in practical situations, where, for example, high pressures and/or large pipe diameters occur, often the present flow regime is unknown (e.g. Simpson *et al.* 1987).

A striking phenomenon in the existing literature concerning two-phase flow is the lack of theoretical and experimental studies on horizontal gas–liquid pipe flow with small values of the liquid holdup (e.g. $\epsilon_L \leqslant 0.06$). This type of flow may occur, when natural gas of high pressure is transported through pipelines. At certain positions in the pipe traces of liquid can be formed by

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retrograde condensation as a result of pressure decrease. The presence of these traces of liquid may have a marked effect on the gas–liquid frictional pressure gradient. It can be shown (Hamersma & Hart 1987) that in a pipe a liquid holdup of 0.005 may result in a rise of the pressure gradient of up to 30%, as compared with single-phase gas flow.

In spite of this large effect of the presence of liquid on the pressure gradient in gas–liquid pipe flow, this kind of flow has not been studied extensively before. For example, Dukler *et al.* (1964) stated that one source of experimental errors in two-phase flow data, in general, is the low accuracy of low liquid-holdup data and that comparing these data ($0.01 < \epsilon_L < 0.059$) with existing correlations is meaningless. Until now investigations on gas–liquid pipe flow with a small liquid holdup have mostly been restricted to *annular* flow in vertical tubes, such as wetted wall columns (Suzuki 1986) and in horizontal pipes (Willets *et al.* 1987).

For annular gas–liquid flow most of these studies deal with experiments concerning liquid-film thickness and pressure gradient. From these studies it appears that the existence of waves on the liquid-film surface is an important factor for the description of transport phenomena in the gas–liquid system mentioned. Waves are generated by the gas phase flowing over the liquid film. Since amplitude and the length of the waves are dependent on the whole set of physical quantities, occurring in gas–liquid flow (Friedel 1977), the description of the behaviour of these waves and that of the closely related interfacial friction, is difficult. Usually, the interfacial friction factor is derived from friction factors for rough tubes (Gilchrist & Naom 1987), where the "sand roughness" is a measure for the interfacial roughness. This approximation gives a reasonable description of the roughness of the liquid film and is suitable for engineering purposes (Hamersma 1983). In his model for the description of gas–condensate flow in horizontal and inclined pipes Oliemans (1987) incorporated an interfacial friction factor, based on sand roughness. This model was related to the familiar Taitel & Dukler (1976) momentum balance. A restriction of this model is that only the geometry of stratified gas–liquid pipe flow is considered.

Most of the above-mentioned studies do not report experimentally determined low liquid-holdup values. Only Spedding & Chen (1984) have also presented a number of experimentally determined low liquid-holdup data for air–water flow in a horizontal pipe. They described the results of these measurements by the following correlation for the liquid holdup ϵ_L:

$$\epsilon_L = \left[1 + 0.45\left(\frac{u_G}{u_L}\right)^{0.65}\right]^{-1}, \qquad \text{for } \epsilon_L \leq 0.20, \tag{1}$$

where u_G and u_L are the superficial velocities of gas and liquid, respectively.

Hamersma & Hart (1987) reported experimental results for an air–water and an air–water + ethyleneglycol system with low liquid-holdup values. These experimental holdup data could be correlated by the following equation:

$$\epsilon_L = \left[1 + a\left(\frac{u_G}{u_L}\right)^{b}\left(\frac{\rho_G}{\rho_L}\right)^{c}\right]^{-1}, \qquad \text{for } \epsilon_L \leq 0.04, \tag{2}$$

where ρ_G and ρ_L are the densities of gas and liquid, respectively, and

$a = 3.81, \quad b = 2/3, \quad c = 1/3$ for the air–water system,

$a = 3.10, \quad b = 2/3, \quad c = 1/3$ for the air–water + glycol system.

The correlation [1] of Spedding & Chen is a special case of [2].

Our experiments confirm the experimental findings of Spedding & Chen (1984) that considerable deviations from the above correlations occur at relatively low superficial gas velocities ($u_G < 8 \text{ m s}^{-1}$). This may be caused by a change of the flow regime from stratified–wavy to stratified flow. Therefore, we performed theoretical and experimental investigations in a larger range of superficial gas velocities ($5 < u_G < 30 \text{ m s}^{-1}$) and transport properties ($0.9 \leq \eta_L \leq 8.5$ mPa s; $38 \leq \sigma \leq 72$ mPa m) to give a better description of small values of the liquid holdup during gas–liquid flow in a horizontal pipe.

In Section 2 the apparent rough surface (ARS) model will be introduced to calculate the frictional pressure drop ΔP_{TP}, the wetted wall fraction θ and the liquid holdup ϵ_L for horizontal gas–liquid pipe flow with small liquid-holdup values [see also Hart (1988)].

2. MODEL

2.1. *The frictional pressure drop ΔP_{TP}*

Hamersma & Hart (1987) have derived a model for the frictional pressure gradient in the gas phase of isothermal, steady-state gas–liquid flow through horizontal pipes with small liquid-holdup values ($\epsilon_L < 0.04$) in the stratified–wavy, wavy and annular flow regimes. For this kind of flow it was shown that the liquid, flowing along the tube wall, may be considered as a local roughness over a fraction $\theta = \alpha/2\pi$ of the tube wall, see figure 1. The interfacial shear stress τ_i exerted by the gas phase on the liquid film may be considered as shear stress exerted by the gas phase on a rough wall. These assumptions lead to the following correlation for the two-phase pressure drop ΔP_{TP}:

$$\Delta P_{TP} = 4f_{TP}\left(\frac{L}{D}\right)\tfrac{1}{2}\rho_G v_G^2, \qquad \text{for } v_G \gg v_i, \tag{3}$$

where f_{TP} is the two-phase friction factor and v_G is the average axial velocity of the gas phase; $v_G = u_G/(1 - \epsilon_L)$.

If, however, the interface velocity v_i may not be neglected with regard to v_G, [3] changes into (see [A.5] in appendix A with $\beta = 0$):

$$(1 - \epsilon_L)\,\Delta P_{TP} = 4f_{TP}\left(\frac{L}{D}\right)\tfrac{1}{2}\rho_G v_G^2 - 4\theta f_i\left(\frac{L}{D}\right)\tfrac{1}{2}\rho_G(2v_G v_i - v_i^2), \tag{4}$$

where $\theta = \alpha/(2\pi)$ is the wetted fraction of the tube wall and f_i is the interface friction factor. In [3] and [4] the two-phase friction factor f_{TP} is given by

$$f_{TP} = (1 - \theta)\cdot f_G + \theta\cdot f_i. \tag{5}$$

The single-phase friction factor f_G, due to drag exerted by the gas phase on the non-wetted part of the tube wall, can be obtained from (Eck 1973)

$$f_G = \frac{0.07725}{[\log_{10}(\mathrm{Re}_G/7)]^2}, \tag{6}$$

which is valid for $2100 < \mathrm{Re}_G < 10^8$, where

$$\mathrm{Re}_G = \frac{D v_G \rho_G}{\eta_G}. \tag{7}$$

The interfacial friction factor f_i results from drag exerted by the gas phase on a rough surface, i.e. the rippling liquid phase, and is obtained from (Eck 1973)

$$f_i = \frac{0.0625}{\left[\log_{10}\left(\dfrac{15}{\mathrm{Re}_G} + \dfrac{k}{3.715D}\right)\right]^2}, \tag{8}$$

where k/D is the relative sand roughness of the inner tube wall.

Experimental results (Hamersma & Hart 1987) showed that the value of the apparent roughness k of the liquid film, i.e. $k = \delta_{MAX} - \delta_{MIN}$ (see also figure 1), is equal to

$$k \approx 2.3\cdot\delta, \tag{9}$$

in which δ is the average thickness of the liquid film on the *wetted* part θ of the tube wall; δ is assumed to be constant in the tangential direction. Referring to figure 1 it can easily be derived that

$$\delta \approx \frac{\epsilon_L D}{4\theta}, \qquad \text{if } \frac{\delta}{D} \ll 1. \tag{10}$$

It must be noted that the correlation of [8] is based on the so-called "sand roughness". This type of roughness tends to be different from the "roughness" in gas–liquid flow. In the latter case

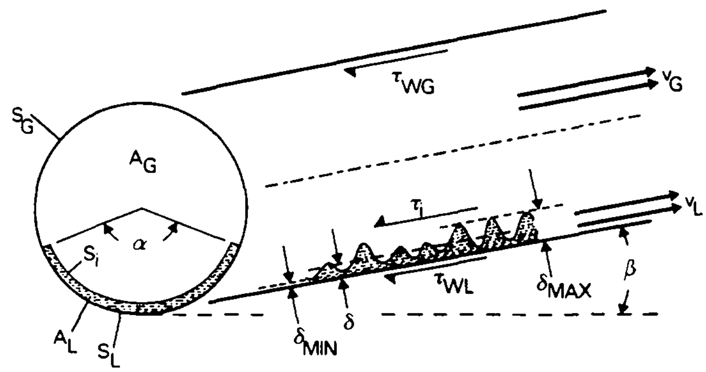

Figure 1. Schematic representation of gas–liquid flow with a small liquid holdup in straight smooth tubes. In the ARS model it is assumed that in a cross-section of the tube the wall is covered over a fraction $\theta = \alpha/(2\pi)$ by a liquid layer of constant average thickness δ in the tangential direction. The value of θ is assumed to be dependent on the kinetic energy of the liquid (see [B.7]). In the axial direction the thickness of the liquid layer varies in the domain $\delta_{MIN} < \delta < \delta_{MAX}$.

rippling occurs. It was recognized (e.g. Schlichting 1980) that ripples, perpendicular to the flow direction, are the cause of a larger friction than that which can be expected from their geometry, based on sand roughness. Therefore, the value of $2.3 \cdot \delta$ for the roughness k of the liquid film is probably larger than the "real roughness" of the liquid film. But since there is no correlation available, which accurately describes the friction factor in pipes with "rippling roughness", [8] in combination with [9] and [10] is used. There is no doubt that the factor 2.3 is dependent on the amplitude and frequency of the waves of the liquid film, and, therefore, also on the flow regime and transport properties of the gas–liquid system.

2.2. *The wetted wall fraction θ*

For the wetted fraction θ of the tube wall during gas–liquid flow with a small liquid holdup in the stratified–wavy flow regime, the following equation was derived in a previous paper (Hamersma & Hart 1987):

$$\theta = \left(\frac{1}{\pi}\right)\arccos(1 - C_1\,\mathrm{Fr}), \tag{11}$$

in which the constant C_1 was determined empirically as $C_1 = 0.66$. Equation [11] is valid for $\mathrm{Fr} \leqslant 2/C_1$. if $\mathrm{Fr} > 2/C_1$ then $\theta = 1$, i.e. annular flow occurs. In [11] the modified Fr is defined by

$$\mathrm{Fr} = \left(\frac{\rho_L}{\Delta\rho}\right)\left(\frac{v_L^2}{gD}\right), \tag{12}$$

where $\Delta\rho = \rho_L - \rho_G$ and g is the acceleration due to gravity.

An implication of [11] is that the value of the wetted wall fraction θ should approach zero for the hypothetical situation that $\mathrm{Fr} \to 0$. From our measurements, however, it appeared that for small values of Fr ($\mathrm{Fr} < 0.25$) a certain minimum value of the wetted wall fraction $\theta = \theta_0$ is held by the liquid phase. Since for small values of Fr the discrepancy between the experimentally determined values of the wetted wall fraction and values calculated with [11] could be >300%, a more accurate description of θ is required. In appendix B a model has been derived, resulting in the following correlation:

$$\theta = \theta_0 + C_2\,\mathrm{Fr}^{0.58}, \tag{13}$$

where the constant C_2 has been obtained from experiments ($C_2 = 0.26$; see section 4.1). This equation gives a better description of the experimental results (see section 4.1) than [11] and is valid for $\theta \leqslant 1$ or $\mathrm{Fr} \leqslant [(1 - \theta_0)/C_2]^{1.71}$. If $\mathrm{Fr} > [(1 - \theta_0)/C_2]^{1.71}$, annular flow occurs and $\theta = 1$. The value of θ_0, which can be regarded as the minimum value of θ if $\mathrm{Fr} \to 0$, has also been derived in appendix B and can be approximated by

$$\theta_0 \approx 0.52\epsilon_L^{0.374}. \tag{14}$$

2.3. *The liquid holdup ϵ_L*

Butterworth (1975) found that most of the liquid- or gas-holdup correlations in the literature can be rewritten into the following equation:

$$\frac{\epsilon_L}{1-\epsilon_L} = a\left(\frac{u_L}{u_G}\right)^b \left(\frac{\rho_L}{\rho_G}\right)^c \left(\frac{\eta_L}{\eta_G}\right)^d, \qquad [15]$$

where η_G and η_L are the dynamic viscosities of gas and liquid, respectively. The values of a, b, c and d in [15] appear to be constant for a limiting range of values of the superficial velocities u_G and u_L and the transport properties. Equations [1] and [2] are special cases of [15].

From the force balance under steady-state conditions we derived the following equation describing the liquid holdup in the stratified, wavy and annular flow regimes (see appendix C):

$$\frac{\epsilon_L}{1-\epsilon_L} = \frac{u_L}{u_G}\left[1+\left(\frac{f_L \rho_L}{f_i \rho_G}\right)^{1/2}\right], \qquad \text{for } \epsilon_L \leqslant 0.06, \qquad [16]$$

where f_L is the friction factor referring to the shear stress between the liquid film and tube wall. In section 4.2 it will be shown on the basis of experimental results that the ratio f_L/f_i is a function of the superficial Reynolds number Re_{SL} only ($\mathrm{Re}_{SL} = D u_L \rho_L/\eta_L$).

3. TEST FACILITY

3.1. *Test section and accessories*

A schematic representation of the test facility is given in figure 2. The experiments were carried out in a straight, horizontal copper tube (12) with i.d. 51 mm and length approx $\simeq 17$ m, including a 1.4 m glass section (13). The largest deviation of the tube from the horizontal was about 0.05°.

Air saturated with water was supplied from a water-ring compressor (1), providing a relative humidity of the air of >90%. This is essential in measuring low liquid-holdup values, because evaporation of part of the liquid in the pipe may have a marked effect on the measured liquid-holdup value. The air flow rate was controlled by one of the two rotameters (3) covering the ranges 2.77–19.4 and 11.1–83.3 $\mathrm{dm^3\,s^{-1}}$, respectively. The liquid was supplied from a vessel (7), using a rotary displacement pump (8), to one of the three rotameters (11), with different measuring ranges. The liquid was injected through an injection port (6) at the bottom of the pipe.

At a distance of about 4 m (80 pipe dia) from the injection point a precision glass tube (13) was inserted, which was provided with an angle gauge to measure the angle over which the tube was wetted by the liquid phase with an accuracy of 2°. Two pairs of pressure taps (14) were located

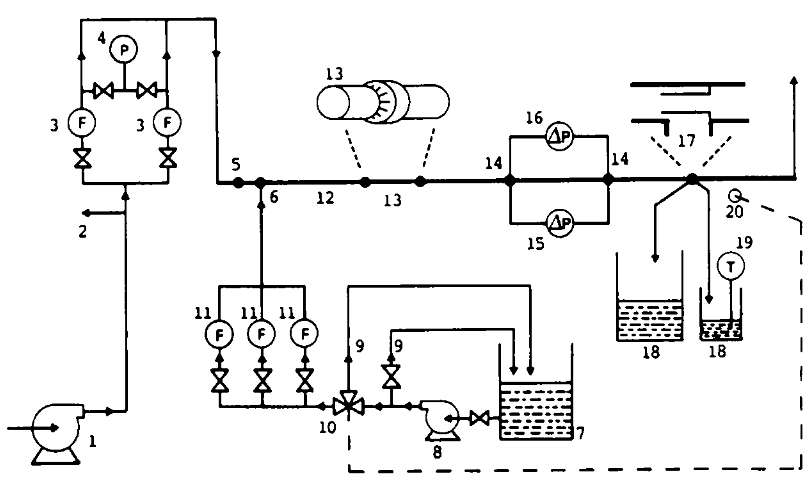

Figure 2. Test facility: 1, water-ring compressor; 2, gas outlet; 3, gas flowmeter; 4, pressure indicator; 5, sieve plate; 6, liquid injection; 7, liquid storage tank; 8, rotary displacement pump; 9, liquid bypass; 10, magnetic three-way valve; 11, liquid flowmeter; 12, copper tube (L = 17 m, D = 0.051 m); 13, Glass tube with angle gauge; 14, pressure taps; 15, water manometer; 16, micromanometer; 17, gas–liquid separator; 18, liquid receiving bins; 19, temperature indicator; 20, switch of magnetic ventile.

at distances of 6 and 11 m (120 and 220 pipe dia) from the liquid-injection point. One pair of pressure taps, dia 1 mm, was located at the top of the tube, to measure the pressure gradient over the gas phase with a precision micromanometer (16), having an accuracy of 0.4–4 Pa, depending on the slope of the manometer tube. The other pair of pressure taps, located at the bottom of the tube, was used in combination with a water manometer (15) to measure the pressure gradient across the liquid phase. During our experiments no significant differences were found between values of the time-averaged static-pressure gradient obtained with the two methods.

At the end of the tube the liquid and gas were separated by means of a gas–liquid separator (17). After shutting off the liquid injection with a quick-closing magnetic ventile (10) and removing the remaining liquid from the tube with the saturated air flow, the liquid holdup was determined by weighing. The repeatability and reproducibility of this method (>99% for $\epsilon_L > 0.01$) enabled us to measure minimum liquid-holdup values of 0.0012 with an accuracy of about 96%.

3.2. Fluid systems and test conditions

Measurements were carried out at superficial air velocities ranging from about 5 to 30 m s^{-1}, superficial liquid velocities ranging from $0.25 \cdot 10^{-3}$ to $80 \cdot 10^{-3}$ m s^{-1} and temperatures in the range 15–25°C. Flow regimes observed during the experiments were stratified, stratified–wavy and annular flow, as has been indicated in the flow pattern map of Mandhane *et al.* (1974) given in figure 3.

For the determination of the effect of the interfacial tension σ on the wetted wall fraction θ, the liquid holdup ϵ_L and the frictional pressure gradient $\Delta P_{TP}/L$, water was used in combination with a surface active agent (Tween 80, obtained from J. T. Baker Chemicals B. V.) to diminish the interfacial tension without changing other transport properties. Care was taken not to exceed the concentration where foaming started to occur.

The effect of the liquid viscosity on the liquid holdup ϵ_L, the wetted wall fraction θ and the two-phase pressure gradient $\Delta P_{TP}/L$ was investigated for several air–water + ethyleneglycol mixtures. The transport properties of the fluid systems used are listed in table 1.

4. EXPERIMENTAL RESULTS

4.1. The wetted fraction θ of the tube wall

Figure 4 shows the experimentally determined values of the wetted wall fraction θ as a function of the modified Fr [$\mathrm{Fr} = (\rho_L/\Delta\rho)v_L^2/gD$] for the air–water and an air–water + Tween

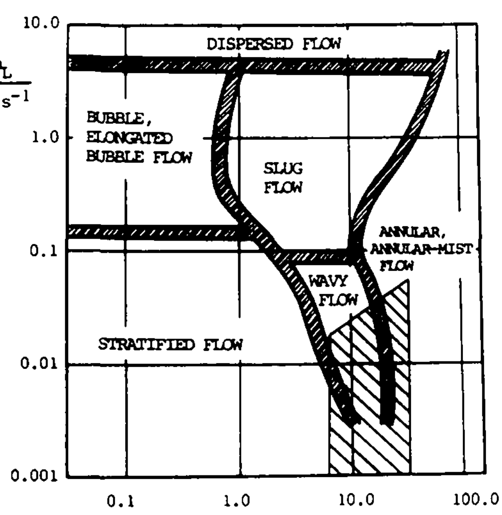

Figure 3. Mandhane *et al.*'s (1974) flow pattern map for horizontal gas–liquid pipe flow. The hatched area covers the region treated in this paper.

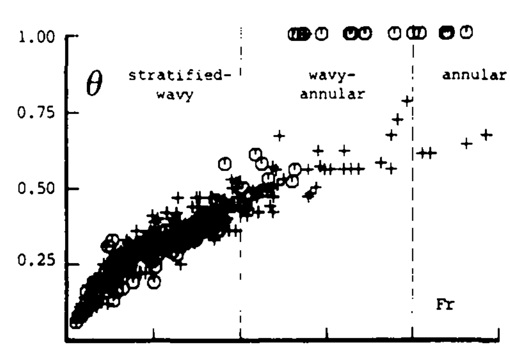

Figure 4. Experimentally determined values of the wetted wall fraction θ as a function of the modified Fr: +, air–water system (σ = 72 mN m^{-1}); ○, air–water + 0.11 wt% Tween 80 system (σ = 38 mN m^{-1}).

Table 1. Transport properties of the fluid systems used at 20°C

Fluid system	Dynamic viscosity (mPa s)	Surface tension ($mN\,m^{-1}$)	Density ($kg\,m^{-3}$)
Humid air	0.0178	—	1.23
Water	1.00	72	998
Water + 0.008 wt% Tween 80	1.00	60	998
Water + 0.020 wt% Tween 80	1.00	47	998
Water + 0.110 wt% Tween 80	1.00	38	998
Water + 11.5 wt% glycol	1.13	65	1011
Water + 25 wt% glycol	1.76	63	1025
Water + 35 wt% glycol	2.47	61	1039
Water + 80 wt% glycol	8.50	56	1091

($\sigma = 38\ mN\,m^{-1}$) system. From this figure it appears that no significant effect of the surface tension σ on the value of the wetted wall fraction θ can be observed for $\theta < 0.6$.

The value of Fr_{crit}, i.e. the Fr where annular flow starts to occur, decreases slightly with decreasing surface tension σ. There is however a poor correlation between Fr_{crit} and σ. Therefore, it shall be stated that for $Fr < 2$ a decrease in the surface tension has little effect on the value of the wetted wall fraction. For the region $Fr < 2$, it is unlikely that annular flow occurs in gas–liquid flow through a horizontal, straight tube. For the region $2 < Fr < 4$, the boundary between wavy and annular flow depends on the value of the surface tension. For $Fr > 4$, annular flow may be expected, i.e. $\theta = 1$.

From experiments with the air–water and air–water + glycol mixtures the value of the constant C_2 in [13] turned out to have a best-fit value of $C_2 = 0.26$. In figure 5, the experimentally determined values of the wetted wall fraction have been plotted as a function of the values calculated with [13] and [14]. An average relative deviation (RD) of about 19% is obtained if [13] and [14] are used (see also section 4.4). This is considerably better than the average RD of about 80% if [11] is applied. Thus, the present model gives a better description of the wetted wall fraction θ than that published previously (Hamersma & Hart 1987).

4.2. The liquid holdup ϵ_L

According to the correlation of section 2.3, the value of the liquid holdup in horizontal gas–liquid pipe flow can be obtained from [16]:

$$\frac{\epsilon_L}{1-\epsilon_L} = \frac{u_L}{u_G}\left[1 + \left(\frac{f_L \rho_L}{f_i \rho_G}\right)^{1/2}\right], \qquad \text{for } \epsilon_L \leqslant 0.06.$$

Rearranging [16], we obtain

$$\frac{f_L}{f_i} = \left(\frac{u_G}{u_L} \cdot \frac{\epsilon_L}{1-\epsilon_L} - 1\right)^2 \cdot \frac{\rho_G}{\rho_L}. \tag{17}$$

Our experimental results showed that there is a pronounced correlation between the ratio f_L/f_i, calculated from [17], and the superficial Reynolds number of the liquid phase Re_{SL},

$$\frac{f_L}{f_i} = 108\ Re_{SL}^{-0.726}, \tag{18}$$

in which 108 and -0.726 are empirical constants. This equation is much simpler than correlations obtained by using the separate friction factors f_L and f_i, an approach which is generally accepted in the literature (Lockhart & Martinelli 1949; Taitel & Dukler 1976; Chen & Spedding 1983; Oliemans 1987).

Substitution of [18] into [16] gives the following correlation containing two empirical constants:

$$\frac{\epsilon_L}{1-\epsilon_L} = \frac{u_L}{u_G}\left\{1 + \left[10.4\ Re_{SL}^{-0.363}\left(\frac{\rho_L}{\rho_G}\right)^{1/2}\right]\right\}. \tag{19}$$

Figure 6 shows that for five different gas–liquid systems a good agreement is obtained between the experimentally determined values of the liquid holdup and the values calculated with [19].

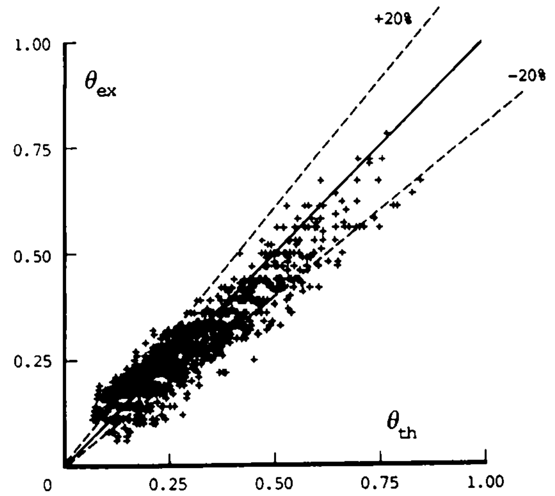

Figure 5. Experimentally determined values of the wetted wall fraction θ for the air–water and air–water + ethylene-glycol systems as a function of the values calculated with [13], where $C_2 = 0.26$.

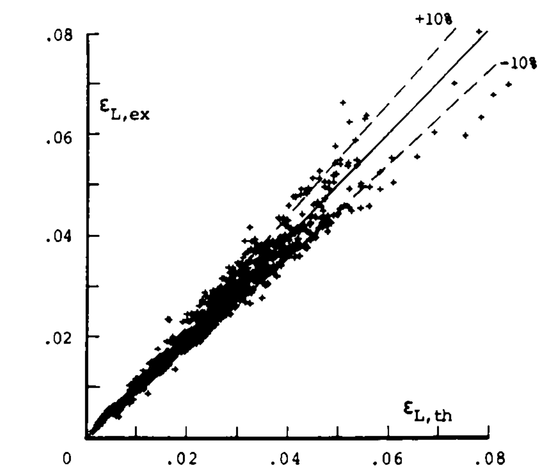

Figure 6. Experimentally determined values of the liquid holdup ϵ_L for the air–water and four different air–water + ethyleneglycol systems as a function of values calculated with [19]. The transport properties are listed in table 1.

Additional measurements showed that the liquid-holdup correlation [19] also holds for measurements concerning low liquid-holdup values performed at our laboratory in a 15 mm i.d. tube on an air–water system. This result is important because it demonstrates that the effect of the tube diameter on the value of the liquid holdup is also included in [19].

Besides the comparison of literature correlations, presented in a previous paper (Hamersma & Hart 1987), we have compared Eaton's correlation (Eaton *et al.* 1967) and correlation [19] in the present paper using the data of Andrews (1966; the "Eaton data"), Minami (1983) and our experimental results. It was found that:

(1) The holdup values predicted with [19] agree with
- our experimental results ($0.0012 < \epsilon_L < 0.06$) within average RD limits, RD(ϵ_L) (see also section 4.4), of about 10%,
- Andrews's experimental results (natural gas–water system; $D = 0.0525$ m; $15 < P < 40$ bar; $0.006 < \epsilon_L < 0.75$) within average RD limits of about 17% (see figure 7),
- Minami's experimental results (air–kerosene system; $D = 0.078$ m; $3 < P < 7$ bar; $0.01 < \epsilon_L < 0.43$) within average RD limits of about 23% (see figure 7),

(2) The holdup values predicted with Eaton's multi-parameter correlation (Eaton *et al.* 1967)
- agree with Andrews's holdup data ($0.006 < \epsilon_L < 0.75$) within average RD limits of about 20%,
- differ considerably from our experimental results ($0.0012 < \epsilon_L < 0.06$); an average RD of about 400% has been obtained.

Further, our measurements showed that the value of the liquid holdup is *not* affected by the value of the interfacial tension $38 < \sigma < 72$ mPa m. This holds only for conditions where the liquid entrainment is negligible. Willets *et al.* (1987) showed that above a critical liquid-film flow rate the liquid entrainment is strongly dependent on the value of the interfacial tension.

4.3. *The pressure gradient $\Delta P_{TP}/L$*

The value of the two-phase pressure gradient $\Delta P_{TP}/L$ can be obtained with the ARS model according to the procedure given in appendix D.

Figure 8 shows experimentally determined values of the two-phase frictional pressure gradient $\Delta P_{TP}/L$ for air–water and air–water + etyleneglycol systems as a function of the values calculated according to the above-mentioned procedure after substitution of the calculated values of the liquid

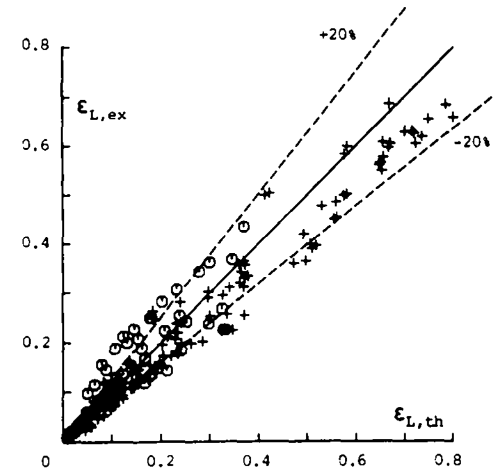

Figure 7. Experimentally determined literature data of the liquid holdup ϵ_L for the natural gas–water (+; Andrews 1966) and air-kerosene (O; Minami 1983) systems as a function of values calculated with [19] of the ARS model.

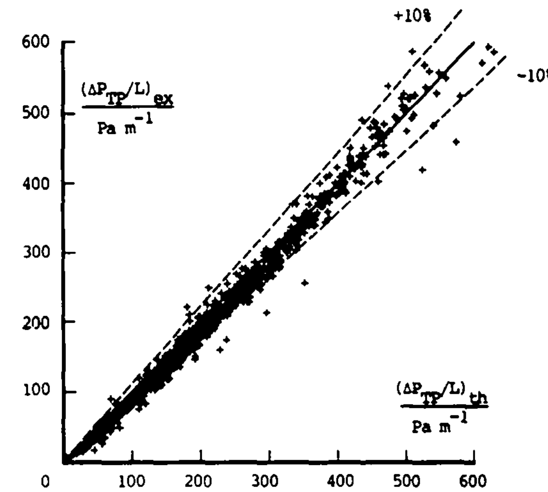

Figure 8. Experimentally determined values of the pressure gradient $\Delta P_{TP}/L$ for the air-water and four different air-water + ethyleneglycol systems as a function of values calculated with the ARS model (see appendix D).

holdup ϵ_L, [19], and the wetted wall fraction θ ([13] and [14] and $C_2 = 0.26$). It can be seen that a good agreement is found between predicted and measured values of the pressure gradient.

Adding surface-active agents to the water has some effect on experimentally determined values of the pressure gradient compared with experimental results obtained with the air-water system. As mentioned in the introduction, the existence of waves in gas-liquid flow is of importance for the pressure gradient of the flowing two-phase system. It is known (Friedel 1977) that a decrease in the surface tension of the flowing gas-liquid system promotes the formation and magnification of waves on a liquid surface. Therefore, it can be expected that the "roughness" k ($k = \delta_{MAX} - \delta_{MIN}$) of the liquid film and, consequently, the pressure gradient are increased. This behaviour is verified in figure 9. In this figure a comparison has been made between measurements on the air-water

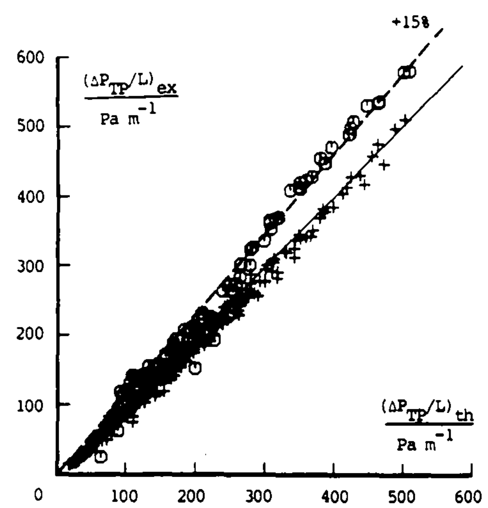

Figure 9. Experimentally determined values of the pressure gradient $\Delta P_{TP}/L$ as a function of the values calculated with the ARS model (see appendix D): +, air-water system ($\sigma = 72\ \mathrm{mN\ m^{-1}}$); O, air-water + 0.11 wt% Tween 80 system ($\sigma = 38\ \mathrm{mN\ m^{-1}}$).

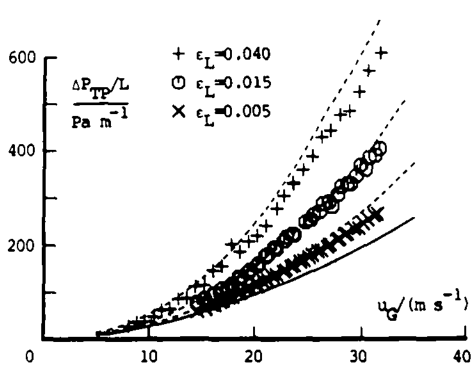

Figure 10. Pressure gradient values $\Delta P_{TP}/L$ during air-water flow in a horizontal tube (dia 51 mm) as a function of the superficial gas velocity u_G for different values of the liquid holdup ϵ_L. The marked points refer to experimentally determined values of the pressure gradient. The solid line represents the calculated pressure gradient in a horizontal straight smooth tube during single-phase air flow. The dashed lines have been calculated with the ARS model (see Appendix D) using average values of the transport properties for the air-water system ($\rho_G = 1.20\ \mathrm{kg\ m^{-3}}$; $\rho_L = 998\ \mathrm{kg\ m^{-3}}$; $\eta_G = 1.78 \cdot 10^{-5}\ \mathrm{Pa\ s}$; $\eta_L = 1.02 \cdot 10^{-3}\ \mathrm{Pa\ s}$).

system and the air–water + 0.11 wt% Tween ($\sigma = 38\ \mathrm{mN\ m^{-1}}$) system. An increase of about 15% in the experimentally determined pressure gradient is observed if the surface tension is decreased by about 50%. Since the use of surface-active agents does not permit quantitative enunciations for the system considered, we will only remark on the phenomenon.

The large increase in the pressure gradient due to the presence of small quantities of liquid, as compared with single-phase gas flow, is clearly demonstrated in figure 10 for the air–water system. From figure 10 it is clear that the presence of small quantities of liquid has significant consequences for the operation of natural gas pipelines. Therefore, an accurate prediction of the liquid holdup and the pressure gradient is important in pipeline design, especially when "wet-gas" systems may be expected.

The results of pressure-drop measurements published in the literature generally refer to conditions which do not cover our area of interest ($0 < \epsilon_L < 0.06$; stratified, wavy and annular flow regimes). For the "Eaton data" (Andrews 1966), for example, only *two* of the 130 pressure-drop measurements refer to stratified, wavy or annular flow with holdup values $\epsilon_L < 0.06$. The two pressure-drop values referring to the natural gas–water system agree well with the values calculated with the ARS model. Other experimental pressure-drop values in the Eaton data refer to larger holdup values and/or slug flow and mist flow regimes and, consequently, these values of the physical quantities cannot be predicted accurately by the procedure given in appendix D.

4.4. *Accuracy of the proposed models*

For gas–liquid flow in horizontal straight tubes the RDs between experimental data and results obtained with the correlations for the wetted wall fraction θ, [13], the liquid holdup ϵ_L, [20], and the two-phase pressure gradient $\Delta P_{TP}/L$, [4], are given in table 2. The respective RDs are defined by

$$\mathrm{RD}(\theta) = \frac{1}{N}\sum^{N}\frac{|\theta_{ex} - \theta_{th}|}{\theta_{th}}\cdot 100\%, \qquad [20]$$

$$\mathrm{RD}(\epsilon_L) = \frac{1}{N}\sum^{N}\frac{|\epsilon_{L,ex} - \epsilon_{L,th}|}{\epsilon_{L,th}}\cdot 100\% \qquad [21]$$

and

$$\mathrm{RD}\left(\frac{\Delta P_{TP}}{L}\right) = \frac{1}{N}\sum^{N}\frac{\left|\left(\frac{\Delta P_{TP,ex}}{L}\right) - \left(\frac{\Delta P_{TP,th}}{L}\right)\right|}{\left(\frac{\Delta P_{TP,th}}{L}\right)}\cdot 100\%, \qquad [22]$$

in which N is the number of experiments.

5. CONCLUSIONS

1. The ARS model developed in the present paper for predicting low values of the liquid holdup ϵ_L and values of the frictional pressure gradient $\Delta P_{TP}/L$ during gas–liquid pipe flow has been

Table 2. The RD between experimental data and results calculated with correlations for θ, ϵ_L and $\Delta P_{TP}/L$ of the ARS model (the number of observations is given in parentheses)

Gas–liquid systems used	RD(θ) (%)	RD(ϵ_L) (%)	RD($\Delta P_{TP}/L$) (%)
Air–water (638)	15.6	8.4	9.2
Air–water + glycol 11.5 wt% (135)	19.7	5.5	11.7
Air–water + glycol 25 wt% (123)	14.0	6.0	8.6
Air–water + glycol 35 wt% (254)	16.1	8.8	10.3
Air–water + glycol 80 wt% (68)	38.0	10.0	7.5
Air–water + Tween, all (303)	26.6	8.4	11.5
All data (1521)	20.7	8.1	9.5

verified with experimental data. These data refer to steady-state gas–liquid flow through horizontal straight smooth tubes and have been obtained both from the literature (Andrews 1966; Minami 1983) and from our experiments covering the domain $0 < \epsilon_L \leqslant 0.06$ and different ranges of flow rates ($5 \leqslant u_G \leqslant 30\ \mathrm{m\,s^{-1}}$; $0.25 \cdot 10^{-3} \leqslant u_L \leqslant 80 \cdot 10^{-3}\ \mathrm{m\,s^{-1}}$) and transport properties ($0.9 \leqslant \eta_L \leqslant 8.5$ mPa s; $38 \leqslant \sigma \leqslant 72$ mPa m).

2. In the domain $0 < \epsilon_L < 0.06$, liquid-holdup values predicted with [19] of the ARS model differ < 10% (see table 2) from experimental liquid-holdup values occurring in co-current steady-state gas–liquid flow through horizontal straight smooth tubes.
3. Frictional pressure gradients in co-current gas–liquid flow with a small liquid holdup ($0 < \epsilon_L < 0.06$) through straight horizontal smooth tubes can be calculated using the procedure in appendix D ($\mathrm{RD}(\Delta P_{TP}) < 12\%$; see table 2).
4. It was found experimentally that under the above-mentioned conditions a decrease in the interfacial tension σ resulted in:

—no effect on the value of the liquid holdup;
—no effect on the value of the wetted wall fraction θ for $\theta < 0.6$;
—a slight increase in both the rippling of the liquid film and the pressure gradient;
—an earlier transition from wavy to annular flow for $\theta > 0.6$.

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APPENDIX A

Frictional Pressure Drop ΔP_{TP} During Co-current Steady-state Gas–Liquid Pipe Flow with Small Liquid-holdup Values ($0 < \epsilon_L < 0.06$)

In the steady-state, the following balance of forces acting on the gas phase holds (see also figure 1):

$$-A_G\left(\frac{dP_{TP}}{dL}\right)_G - \tau_{WG}S_G - \tau_i S_i - \rho_G A_G g \sin(\beta) = 0, \qquad [A.1]$$

where A refers to the cross-sectional area, τ to the shear stress, S to the perimeter and β to the upwardly inclined angle. The indices G and i refer to gas phase and interface, respectively.

Substituting the relations of [C.7], given in appendix C, and assuming $\delta/D \ll 1$, [A.1] results in

$$\frac{dP_{TP}}{dL} = \tfrac{1}{2}f_G\rho_G v_G^2\frac{4(1-\theta)}{\epsilon_G D} + \tfrac{1}{2}f_i\rho_G(v_G - v_i)^2\frac{4\theta}{\epsilon_G D} + \rho_G g \sin(\beta). \qquad [A.2]$$

If the frictional pressure gradient is constant, [A.2] results in

$$\epsilon_G\,\Delta P_{TP} = 4f_{TP}\left(\frac{L}{D}\right)\tfrac{1}{2}\rho_G v_G^2 - 4\theta f_i\left(\frac{L}{D}\right)\tfrac{1}{2}\rho_G(2v_G v_i - v_i^2) + \epsilon_G L\rho_G g \sin(\beta), \qquad [A.3]$$

where

$$f_{TP} = (1-\theta)f_G + \theta f_i. \qquad [A.4]$$

It is often assumed that $v_i \approx v_L$ if the liquid flow is turbulent, and $v_i \approx 2v_L$ if the liquid flow is laminar (e.g. Oliemans 1987). Since it is rather difficult to establish whether the liquid flow is laminar or turbulent we have assumed that $v_i \approx v_L$. For horizontal pipe flow ($\beta = 0$), [A.3] becomes

$$\epsilon_G\,\Delta P_{TP} = 4f_{TP}\left(\frac{L}{D}\right)\tfrac{1}{2}\rho_G v_G^2 - 4\theta f_i\left(\frac{L}{D}\right)\tfrac{1}{2}\rho_G(2v_G v_L - v_L^2). \qquad [A.5]$$

If $v_G \gg v_L$ and $\epsilon_L \leqslant 0.06$, we obtain for horizontal pipe flow:

$$\Delta P_{TP} = 4f_{TP}\left(\frac{L}{D}\right)\tfrac{1}{2}\rho_G v_G^2. \qquad [A.6]$$

APPENDIX B

Wetted Wall Fraction θ During Co-current Steady-state Gas–Liquid Flow in a Horizontal Pipe with Small Liquid-holdup Values ($0 < \epsilon_L < 0.06$)

Consider the situation sketched in figure B1. In cylindrical coordinates the distance $\overline{MZ}$ from the centre M of the tube to the centre of gravity Z of the liquid phase can be obtained from

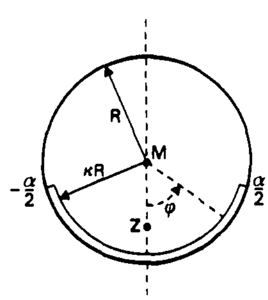

Figure B1. Schematic representation of the cross-section of the geometry considered for gas–liquid pipe flow with small liquid-holdup values. M is the centre of the tube.

$$\overline{\mathrm{MZ}}=\frac{\displaystyle\int_{\kappa R}^{R}\int_{-\alpha/2}^{\alpha/2} r[r\cos(\varphi)]\,\mathrm{d}\varphi\,\mathrm{d}r}{\displaystyle\int_{\kappa R}^{R}\int_{-\alpha/2}^{\alpha/2} r\,\mathrm{d}\varphi\,\mathrm{d}r}, \qquad [\mathrm{B.1}]$$

where r is the radial distance in cylindrical coordinates. The result of [B.1] is

$$\frac{\overline{\mathrm{MZ}}}{R}=\frac{4(1-\kappa^3)\sin\left(\frac{\alpha}{2}\right)}{3(1-\kappa^2)\alpha}. \qquad [\mathrm{B.2}]$$

With the definition $\theta=\alpha/(2\pi)$ and the assumption $\epsilon_L=\theta(1-\kappa^2)$, [B.2] can be written as

$$\frac{\overline{\mathrm{MZ}}}{R}\equiv\xi=\frac{2}{3\pi\epsilon_L}\left[1-\left(1-\frac{\epsilon_L}{\theta}\right)^{3/2}\right]\sin(\pi\theta). \qquad [\mathrm{B.3}]$$

For the situation where annular flow occurs in the pipe ($\theta=1$), it follows from [B.3] that $\xi=0$ or, in other words, the centre Z of gravity of the liquid coincides with the centre M of the tube. This is in agreement with what is to be expected.

From [B.3] it can be derived that for a fixed value of ϵ_L a certain maximum value of ξ exists, i.e. $\xi_0\equiv\overline{\mathrm{MZ}_0}/R$. At $\xi=\xi_0$ the potential energy of the liquid has a minimum value. The value of ξ_0 at a constant value of ϵ_L can be obtained from

$$\frac{\mathrm{d}\xi}{\mathrm{d}\theta}=0. \qquad [\mathrm{B.4}]$$

Combination of [B.3] and [B.4] results in the following equation, which describes the relation between values of the wetted wall fraction where $\xi=\xi_0$ (denoted as θ_0) and the liquid holdup ϵ_L:

$$\tan(\pi\theta_0)=\frac{2\pi\theta_0^2\left[1-\left(1-\frac{\epsilon_L}{\theta_0}\right)^{3/2}\right]}{3\epsilon_L\left(1-\frac{\epsilon_L}{\theta_0}\right)^{1/2}}. \qquad [\mathrm{B.5}]$$

Solving [B.5] iteratively for θ_0 and substituting this value into [B.3] gives the value of ξ_0 for a fixed value of ϵ_L. Thus, the variable θ_0 represents values of θ where the centre of gravity Z of the liquid is located at the maximum distance from the centre M of the tube, or at the smallest distance from the bottom of the tube, provided that the liquid has a geometry as indicated in figure B1. Therefore, θ_0 can be interpreted as a minimum value of θ for a certain value of the liquid holdup ϵ_L. In other words, if the liquid occupies a fraction θ_0 of the tube wall, the liquid has minimal potential energy.

The increase in potential energy of the liquid phase per unit of liquid volume ΔE_p as a result of a shift of the centre of gravity of the liquid from Z_0 to Z can be calculated from

$$\Delta E_p=\Delta\rho g|\mathrm{MZ}-\mathrm{MZ}_0|=V_L\Delta\rho g R(\xi_0-\xi). \qquad [\mathrm{B.6}]$$

Assuming that a fraction C_K of the kinetic energy per unit of liquid volume $E_{K,L}$ is used to account for the increase in ΔE_p, we obtain

$$C_K E_{K,L}=\Delta\rho g R(\xi_0-\xi). \qquad [\mathrm{B.7}]$$

Substituting the kinetic energy per unit of liquid volume into [B.7] results in

$$C_K\tfrac{1}{2}\rho_L v_L^2=\Delta\rho g R(\xi_0-\xi). \qquad [\mathrm{B.8}]$$

This equation can be rewritten as

$$C_K\,\mathrm{Fr}=\xi_0-\xi, \qquad [\mathrm{B.9}]$$

in which

$$\mathrm{Fr}=\left(\frac{\rho_L}{\Delta\rho}\right)\frac{v_L^2}{(gD)}. \qquad [\mathrm{B.10}]$$

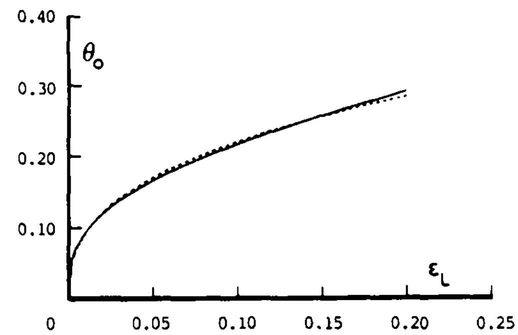

Figure B2. The minimum value of the wetted wall fraction θ_0 as a function of ϵ_L: the dashed line refers to [B.5]; the solid line represents the approximation given by [B.11].

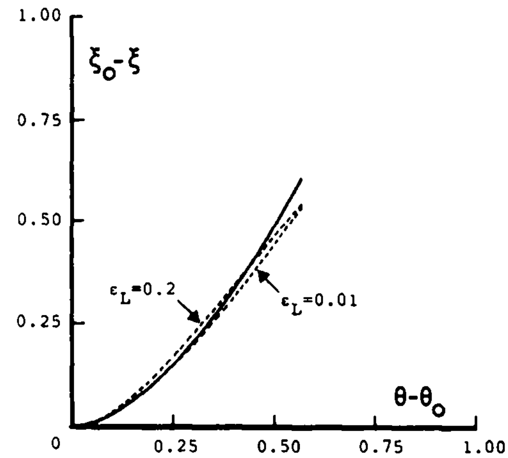

Figure B3. The value of $\xi_0 - \xi$ as a function of $\theta - \theta_0$: the dashed lines have been calculated with [B.3] and [B.5]; the solid line represents the approximation given by [B.12].

The value of C_K has to be obtained from experiments. If the value of the liquid holdup ϵ_L is known, the values of θ_0 [B.5], ξ_0 ([B.3] and $\theta = \theta_0$) and Fr [B.10] can be calculated. Then the value of ξ can be obtained from [B.9] and, finally, the value of the wetted wall fraction θ from [B.3]. However, it must be clear that [B.5], for the determination of θ_0, and [B.3], for the determination of θ, are difficult to handle. Therefore approximations will be made for each of these quantities.

For $0 \leqslant \epsilon_L \leqslant 0.20$, [B.5] may be approximated by the following equation, as shown in figure B2:

$$\theta_0 = 0.52\epsilon_L^{0.374}. \qquad [B.11]$$

It can be verified that the quantity $\xi_0 - \xi$ is a function of $\theta - \theta_0$, which is almost independent of ϵ_L for $0 < \epsilon_L \leqslant 0.20$. Furthermore, from the experiments it appeared that $0 \leqslant \theta - \theta_0 < 0.6$. For these intervals, $\xi_0 - \xi$ may be approximated by

$$\xi_0 - \xi \approx 1.6(\theta - \theta_0)^{1.71}. \qquad [B.12]$$

This relation is plotted in figure B3, together with the relation between $\xi_0 - \xi$ and $\theta - \theta_0$, obtained from [B.3] and [B.5]. Combination of [B.9], [B.12] and $C_2 = \{C_K/1.6\}^{0.58}$, results in

$$\theta = \theta_0 + C_2\mathrm{Fr}^{0.58}, \qquad [B.13]$$

in which the value $C_2 = 0.26$ has been obtained experimentally (see section 4.1).

APPENDIX C

Liquid Holdup During Co-current Steady-state Gas–Liquid Pipe Flow with Small Liquid-Holdup Values ($0 < \epsilon_L < 0.06$)

A generally accepted method for the derivation of a liquid-holdup correlation for steady-state gas–liquid flow is solving the force balance over the liquid phase and the gas phase (e.g. Chen & Spedding 1983; Wu *et al.* 1987) (see also figure 1). Pressure losses due to acceleration are neglected. Referring to figure 1, we may state that for a steady-state gas–liquid flow through a horizontal or upwardly inclined straight smooth tube the following force balances hold:

$$-A_L\left(\frac{dP_{TP}}{dL}\right)_L - \tau_{WL}S_L + \tau_i S_i - \rho_L A_L g \sin(\beta) = 0 \qquad [C.1]$$

and

$$-A_G\left(\frac{dP_{TP}}{dL}\right)_G - \tau_{WG}S_G - \tau_i S_i - \rho_G A_G g \sin(\beta) = 0. \qquad [C.2]$$

It is assumed, and also measured (see section 3), that

$$\left(\frac{dP_{TP}}{dL}\right)_L = \left(\frac{dP_{TP}}{dL}\right)_G. \qquad [C.3]$$

Therefore, combination of [C.1] and [C.2], results in

$$\frac{A_L}{A_G} = \frac{\epsilon_L}{\epsilon_G} = \frac{\tau_{WL} S_L - \tau_i S_i + A_L(\rho_L - \rho_G) g \sin(\beta)}{\tau_{WG} S_G + \tau_i S_i}. \qquad [C.4]$$

Adding 1 to the l.h.s of [C.4] and $(\tau_{WG} S_G + \tau_i S_i)/(\tau_{WG} S_G + \tau_i S_i)$ to the r.h.s. results in

$$\frac{\epsilon_L}{\epsilon_G} + 1 = \frac{1}{\epsilon_G} = \frac{\tau_{WL} S_L + \tau_{WG} S_G + A_L(\rho_L - \rho_G) g \sin(\beta)}{\tau_{WG} S_G + \tau_i S_i}. \qquad [C.5]$$

Rewriting [C.5] gives

$$\tau_{WL} S_L + \tau_{WG} S_G\left(1 - \frac{1}{\epsilon_G}\right) - \frac{\tau_i S_i}{\epsilon_G} + A_L(\rho_L - \rho_G) g \sin(\beta) = 0. \qquad [C.6]$$

For the situation of small liquid-holdup values ($0 < \epsilon_L < 0.06$) and flow regimes where the liquid flows along the tube wall (see figure 1), the terms of [C.6] are given by

$$\tau_{WL} = \tfrac{1}{2} f_L \rho_L v_L^2, \quad S_L = \theta \pi D, \quad A_L = \epsilon_L \frac{\pi}{4} D^2, \quad \tau_i = \tfrac{1}{2} f_i \rho_G (v_G - v_i)^2,$$

$$S_i = \theta \pi (D - 2\delta), \quad \tau_{WG} = \tfrac{1}{2} f_G \rho_G v_G^2, \quad S_G = (1-\theta)\pi D, \quad A_G = \epsilon_G \frac{\pi}{4} D^2. \qquad [C.7]$$

For the situation $\delta \ll D$, and taking into account [C.7], [C.6] can be rewritten into

$$\tfrac{1}{2} f_L \rho_L v_L^2 \theta \pi D - \frac{\epsilon_L}{\epsilon_G} \tfrac{1}{2} f_G \rho_G v_G^2 (1-\theta)\pi D - \frac{1}{\epsilon_G} \tfrac{1}{2} f_i \rho_G (v_G - v_i)^2 \theta \pi D + \epsilon_L \frac{\pi}{4} D^2 (\rho_L - \rho_G) g \sin(\beta) = 0. \qquad [C.8]$$

Dividing [C.8] by $\tfrac{1}{2}\theta \pi D f_L \rho_L v_G^2$ and assuming $v_i \approx v_L$, leads to

$$\frac{v_L^2}{v_G^2} = \frac{\epsilon_L}{\epsilon_G} \frac{1-\theta}{\theta} \frac{\rho_G f_G}{\rho_L f_L} + \frac{f_i \rho_G (v_G - v_L)^2}{\epsilon_G f_L \rho_L v_G^2} - \frac{\epsilon_L D}{4\theta} \frac{(\rho_L - \rho_G)}{\rho_L} \frac{g \sin(\beta)}{\tfrac{1}{2} f_L v_G^2}. \qquad [C.9]$$

For small liquid-holdup values and/or annular flow ($\theta = 1$) the first term on the r.h.s. can be neglected. Substitution of $v_L = u_L/\epsilon_L$ and $v_G = u_G/\epsilon_G$, results in

$$\frac{\epsilon_G^2}{\epsilon_L^2} = \frac{1}{\epsilon_G} \frac{f_i}{f_L} \frac{\rho_G}{\rho_L} \frac{(v_G - v_L)^2}{v_G^2} \frac{u_G^2}{u_L^2} - \frac{\epsilon_L D}{4\theta} \frac{(\rho_L - \rho_G)}{\rho_L} \frac{g \sin(\beta)}{\tfrac{1}{2} f_L v_G^2} \frac{u_G^2}{u_L^2}. \qquad [C.10]$$

This equation can be simplified by substitution of $u_G^2/v_G^2 = \epsilon_G^2$ and multiplying by ϵ_L^2. Further, the following modified Fr can be defined:

$$\mathrm{Fr}_\delta = \frac{u_L^2}{\epsilon_L^2 g \delta} \cdot \frac{\rho_L}{\Delta\rho}, \quad \text{in which} \quad \delta = \frac{\epsilon_L D}{4\theta} \qquad (\delta \ll D). \qquad [C.11]$$

Equation [C.10] can be rewritten as

$$\left(\frac{v_G - v_L}{v_L}\right)^2 = \frac{\epsilon_G f_L \rho_L}{f_i \rho_G}\left[1 + \frac{2\sin(\beta)}{f_L \mathrm{Fr}_\delta}\right]. \qquad [C.12]$$

Substituting $v_L = u_L/\epsilon_L$ and $v_G = u_G/\epsilon_G$ in [C.12] and taking the square root gives the following equation for the liquid holdup during gas–liquid flow in straight pipes:

$$\frac{\epsilon_L}{\epsilon_G} = \frac{u_L}{u_G}\left\{1 + \left(\frac{\epsilon_G f_L \rho_L}{f_i \rho_G}\right)^{1/2}\left[1 + \frac{2\sin(\beta)}{f_L \mathrm{Fr}_\delta}\right]^{1/2}\right\}. \quad [C.13]$$

This equation can only be solved by means of an iterative procedure. As a first guess [C.13] may be solved for $\beta = 0$. For horizontal pipe flow ($\beta = 0$) and $\epsilon_L \leqslant 0.06$, [C.13] becomes

$$\frac{\epsilon_L}{1-\epsilon_L} = \frac{u_L}{u_G}\left[1 + \left(\frac{f_L \rho_L}{f_i \rho_G}\right)^{1/2}\right]. \quad [C.14]$$

The ratio f_L/f_i has to be obtained experimentally (see [18]).

APPENDIX D

ARS Model

The procedure for the calculation of the liquid holdup ϵ_L and the pressure drop ΔP_{TP} during horizontal co-current steady-state gas–liquid pipe flow with small liquid-holdup values ($0 < \epsilon_L < 0.06$) is as follows.

1. Calculate the value of the liquid holdup with

$$\frac{\epsilon_L}{1-\epsilon_L} = \frac{u_L}{u_G}\left\{1 + \left[10.4\,\mathrm{Re}_{SL}^{-0.363}\left(\frac{\rho_L}{\rho_G}\right)^{1/2}\right]\right\}. \quad [D.1]$$

2. Determine the values of the average local gas and liquid velocities, respectively:

$$v_G = \frac{u_G}{1-\epsilon_L}; \quad v_L = \frac{u_L}{\epsilon_L} \quad [D.2]$$

3. Calculate the modified Fr and the Re of the gas phase:

$$\mathrm{Fr} = \frac{v_L^2}{gD}\cdot\frac{\rho_L}{\Delta\rho}; \quad \mathrm{Re}_G = \frac{D v_G \rho_G}{\eta_G}. \quad [D.3]$$

4. Calculate the value of the wetted wall fraction θ:

$$\theta = \theta_0 + 0.26\,\mathrm{Fr}^{0.58}, \quad \text{in which} \quad \theta_0 = 0.52\epsilon_L^{0.374}. \quad [D.4]$$

   If $\theta > 1$, take $\theta = 1$.

5. Determine the value of the apparent relative roughness of the liquid film k/D from

$$\frac{k}{D} = 2.3\cdot\left(\frac{\delta}{D}\right) \simeq 2.3\cdot\left(\frac{\epsilon_L}{4\theta}\right). \quad [D.5]$$

6. Calculate the value of the interfacial friction factor from

$$f_i = \frac{0.0625}{\left[\log_{10}\left(\frac{15}{\mathrm{Re}_G} + \frac{k}{3.715D}\right)\right]^2}. \quad [D.6]$$

7. Calculate the value of the single-phase friction factor f. For *smooth* tubes:

$$f_G = \frac{0.07725}{\left[\log_{10}\left(\frac{\mathrm{Re}_G}{7}\right)\right]^2}, \quad [D.7]$$

   which is valid for $2100 < \mathrm{Re}_G < 10^8$. For *rough* tubes [D.6] must be applied with the proper values of the relative pipe roughness, obtained from, for example, the *Chemical Engineers' Handbook* (Perry & Chilton 1973).

8. Calculate the two-phase friction factor f_{TP} from

$$f_{TP} = (1-\theta)\cdot f + \theta\cdot f_i. \quad [D.8]$$

9. Calculate the two-phase pressure drop ΔP_{TP}:

$$\Delta P_{TP} = \left(\frac{1}{1-\epsilon_L}\right) \cdot \left[4f_{TP}\left(\frac{L}{D}\right)\frac{1}{2}\rho_G v_G^2 - 4\theta f_i\left(\frac{L}{D}\right)\frac{1}{2}\rho_G(2v_G v_L - v_L^2)\right]. \qquad \text{[D.9]}$$

10. If $v_G \gg v_L$ and $0 < \epsilon_L < 0.06$, we obtain for horizontal pipe flow:

$$\Delta P_{TP} = 4f_{TP}\left(\frac{L}{D}\right)\frac{1}{2}\rho_G v_G^2. \qquad \text{[D.10]}$$

Application of this procedure results in the values of $\Delta P_{TP}/L$ presented in figures 8 and 9.